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Negative compressibility

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Structures made up from bi-material elements which can exhibit negative properties, in particular negative compressibility (negative bulk modulus, i.e. expand in size when the external pressure is increased and shrink when the external pressure is decreased) are proposed. This anomalous behaviour is confirmed through finite element modelling.

1 Introduction Several studies on systems and materials exhibiting negative properties have been carried out in the past few decades. These include studies on systems exhibiting negative Poisson's ratios [1–7], negative thermal expansion [8–12] as well as other negative effects [13–16]. Recent work has also suggested that negative compressibility (compressibility being the inverse of the bulk modulus), see Fig. 1, a process which is generally assumed to be forbidden by classical thermodynamics [17–19], may in fact be possible [16, 20, 21]. In particular, Baughman et al. showed that some rare crystal phases may expand in one or more dimensions when hydrostatically compressed [16], whilst more recently, Lakes and Wojciechowski have challenged the assumption that the bulk modulus needs to be positive and argue that there exists no convincing interpretation in thermodynamics which states that a *constrained* solid object may not exhibit a negative bulk modulus [21].

Here, we report a novel 2D *unconstrained* porous system which can exhibit isotropic negative compressibility and that can be easily and cheaply constructed from readily available conventional materials, thus confirming that negative compressibility can indeed exist.

2 Concept It has long been known that when two materials having different coefficients of thermal expansion (CTE) are glued together, they will curve when subjected to a change in temperature, as in the case of a bi-metallic strip in a thermostat mechanism. Nevertheless, it is important to note that the same strips will also curve when subjected to a change in pressure, a behaviour which becomes appreciable when the constituent materials have sufficiently different moduli and/or Poisson's ratios, and/or when the pressure change is sufficiently large [22]. This bending is caused by the different strains each material would experience in the free (unbound) state. This mismatch in strains would result in internal stresses which in turn bend the bi-material strip. This is in itself a very important property of bi-material strips which unfortunately is much less publicised than the analogous effect which results when the strips are subjected to temperature changes. Furthermore, such bi-material elements can be used in the construction of smart structures which utilise this effect to exhibit unusual macroscopic properties, including negative compressibility.

One such structure is the porous solid system shown in Fig. 2 which may be regarded as the unit cell of a larger system placed in a fluid (which may be air). To describe the behaviour of this structure, let us first consider a simplified version of it where the system is only constructed from one material (i.e. material 1 = material 2). When this simplified structure is subjected to a change in pressure, it behaves in a conventional manner, in the sense that an increase in the applied hydrostatic pressure will result in a shrinkage of all the components of the system, and *vice versa* if the system is subjected to a decrease in the applied hydrostatic pressure (i.e. positive compressibility which is equal to the intrinsic compressibility of the material).

However, if the system is constructed from two different materials having different properties (i.e. material 1 ≠ material 2) the ligaments in the systems will also bend as a result of the change in pressure, a property which results in a rotation of the 'square nodes'. As a result of this, if the

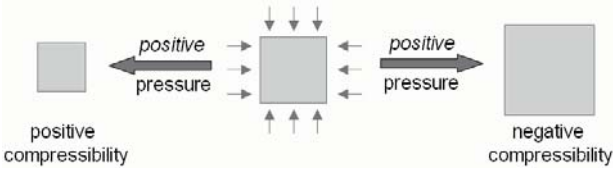

Figure 1 Negative compressibility: unlike conventional systems which shrink when subjected to an increase in pressure, systems with a negative compressibility expand in size when the external pressure increases.

component materials are such that a reduction in pressure results in Material 1 expanding more than Material 2 (for example, by having the Young's modulus of Material 1 smaller than that of Material 2), a reduction in pressure will result in bending of the bi-material ligament in such a way that the centres of the nodes approach each other and *vice versa* for an increase in pressure (i.e. contribute to negative compressibility). In such cases, an overall negative compressibility will be observed if the 'negative contribution' to the compressibility which results from these rotations is larger than the 'positive contribution' which results from the conventional behaviour of the individual components.

This counterintuitive negative behaviour will not be observed if Material 1 is replaced by Material 2 as in this case the curvature of the ligament will result in the centres of squares moving further away from each other when the system is subjected to a reduction in pressure (i.e. positive compressibility). However, such systems are also of interest in view of the fact that the overall positive compressibility of the structure may in fact be larger than that of the constituent materials.

3 Verification In an attempt to verify our hypothesis described above, we performed simulations using the finite element analysis software ANSYS on a typical structure constructed as described in Fig. 2 above with $a = 1$, $b = 12$ and thickness $t_1 = t_2 = 0.05$. This tessellatable unit cell was meshed using Plane 82 elements (an 8 node quadratic element) after the successful completion of a convergence mesh test. Boundary conditions were applied to this model in order to simulate an infinite structure. The constituent material properties were assigned typical values for plastics [23] where Material 1 has a Young's modulus of $E_{S1} = 2.415$ GPa whilst that of Material 2 is $E_{S2} = 4.140$ GPa. In both cases, the Poisson's ratio was assumed to have a value of 0.35. When this system was solved under different pressure conditions, we found that as illustrated in Fig. 3, a positive change in pressure re-

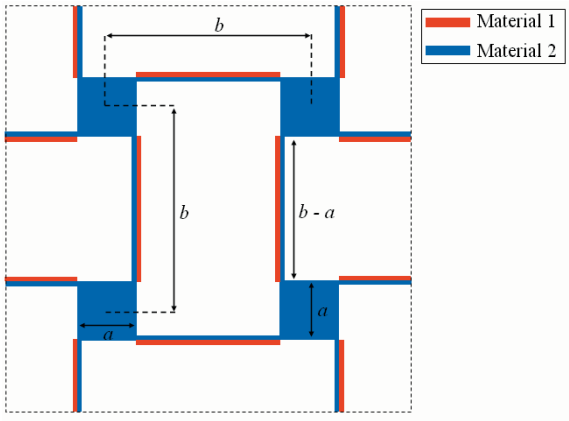

Figure 2 (online colour at: www.pss-rapid.com) Geometry of the structure made up of bi-material ligaments of thickness $t_1 + t_2$ connected to a square node. Note that the squares and part of the ligament thickness (thickness t_1) are made from Material 1 which has a Young's modulus E_{S1}, while the other part of the ligament is made from Material 2 which has thickness t_2 and Young's modulus E_{S2}.

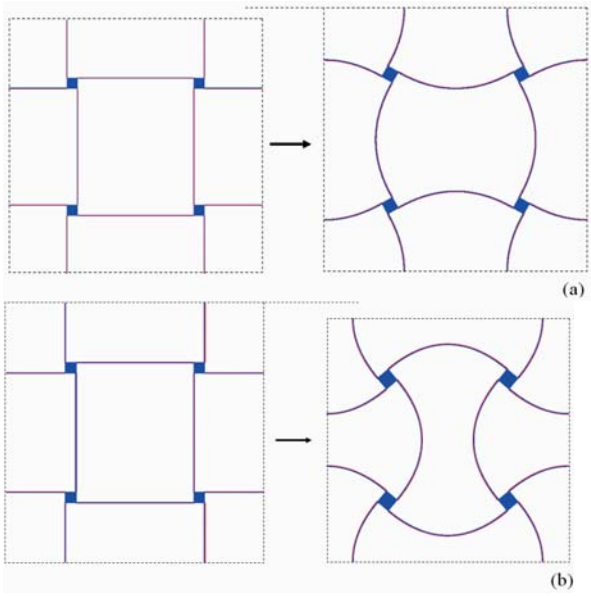

Figure 3 (online colour at: www.pss-rapid.com) System proposed here when subjected to (a) a positive and (b) a negative change in pressure. Parameters used: $a = 1$, $b = 12$, $t_1 = t_2 = 0.05$ of $E_{S1} = 2.415$ GPa, $E_{S2} = 4.140$ GPa, Poisson's ratio of both materials = 0.35.

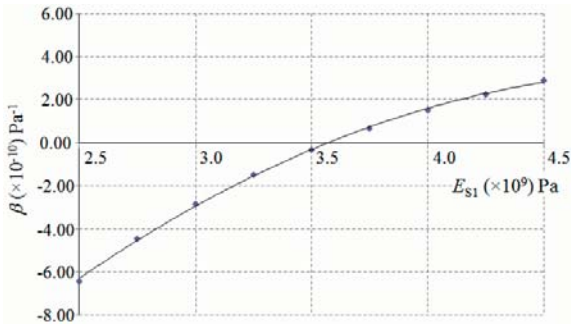

Figure 4 A plot of the compressibility of the system proposed here for different Young's moduli of Material 1. All the other properties were kept constant at $a = 1$, $b = 12$, $t_1 = t_2 = 0.05$, $E_{S2} = 4.140 \times 10^9$ Pa and Poisson's ratio of both materials = 0.35.

sulted in an increase in the overall size of the structure whilst a negative change in pressure resulted in a decrease in size, i.e. the system exhibits negative compressibility.

We also performed additional simulations where the Young's modulus of Material 1 was varied arbitrarily whilst the other parameters were kept constant. As illustrated in Fig. 4, these simulations confirmed that this system can indeed exhibit both negative or positive compressibility, the sign and magnitude being dependent on the relative magnitude of the Young's moduli. In particular, we note that for the systems modelled here, negative compressibility was observed when E_{S1}/E_{S2} is less than ~0.85. All this is very significant as it highlights the fact that the observed value of compressibility can be fine tuned to particular values thus enabling the construction of systems which are tailor made for specific practical applications.

It is important to note that negative compressibility is not the only negative macroscopic property that these systems are expected to exhibit. In fact, apart from being auxetic (i.e. exhibit negative Poisson's ratio), these systems may also undergo negative thermal expansion if Materials 1 and 2 have sufficiently different coefficients of thermal expansion (CTE) with Material 1 having a higher CTE value than Material 2. In fact, it is important to note that any such system which can exhibit negative thermal expansion may also exhibit negative compressibility. In particular we envisage that Lakes's three-dimensional bi-material tetrakaidecahedral structure [9], which was proposed to exhibit negative thermal expansion, may in fact also exhibit negative compressibility if the mechanical properties of the two constituent materials are sufficiently different and their relative position is amenable. Furthermore, it is important to highlight that the properties described here can be shown at any scale of structure, i.e. it is theoretically possible to construct such systems at the microscopic level with the result that the resultant product can be considered as a solid 'material' rather than a 'structure' which exhibits negative compressibility (i.e. negative bulk modulus), negative thermal expansion and/or negative Poisson's ratio (auxetic). This would result in the first solid state material that exhibits such three negative properties simultaneously, thus marking a significant improvement on other proposed systems which may exhibit some, but not all, of these negative properties [1–12, 16].

4 Conclusion We have shown that bi-material ligaments may be used as structural components in the construction of structures which may exhibit, amongst other things, negative compressibility, a property which can potentially be highly desirable in the design of new smart and multifunctional systems. Furthermore, we are providing definite proof (by contradiction) that the compressibility (i.e. the bulk modulus) is not necessarily always positive as most scientists and engineers normally assume.

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